How Long is a Labyrinth

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Introduction

The surprising length of path that can be fitted into the area of a labyrinth is sometimes commented on; e.g. the Chartres Cathedral labyrinth was formerly known as *la Lieue* (the League), and is reported to have taken an hour to cover on hands and knees. The following article gives some quantitative estimates for actual path lengths of various types of labyrinth.

The Classical-type (Cretan) Labyrinth

Various authors have shown how to construct a classical labyrinth using a series of circular arcs with various radii and centres. The geometry of the second-order labyrinth (see footnote 1) is summarized in figure 1. We always start with the central cross, which in turn defines the central square; for the second-order labyrinth the edge of this square is six times the chosen path-width (see table 1 for labyrinths of other orders). We continue by extending the "top" edge of the square left and right, and the two perpendicular edges downwards, to obtain reference lines; and finally, we locate the centre of the labyrinth on the top edge of the square (it is always half a path-width from the upper cross terminal). The construction of the actual walls follows, as shown in figure 1:

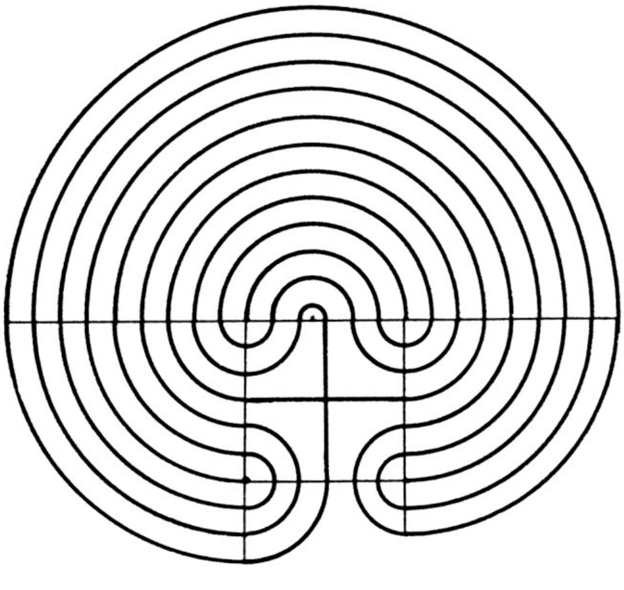


figure 1: the Classical Labyrinth

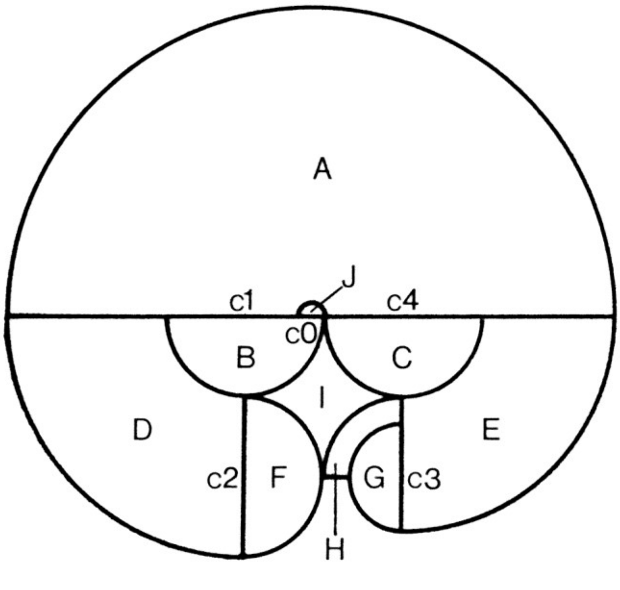


figure 2: the sub-areas of the labyrinth

This process divides the entire area of any classical labyrinth into eight sub-areas, labelled A to H in figure 2 (on the same scale as figure 1). In area A, the labyrinth walls are composed of semicircles centred on c0; areas B and C comprise semicircles centred on c1 and c4 respectively; areas D and E, quarter-circles also centred on c1 and c4 and areas F and G, semicircles centred on c2 and c3 respectively. Area H is the entrance passage and is bounded by quarter-circles centred on c3.

We need to make three assumptions about the path: deciding where precisely it begins; how it behaves in the vicinity of the central cross, where there is tension between the general circular form and the right-angled quadrants; and where precisely it ends. We assume that the actual entrance into the labyrinth takes place at the narrowest "throat" between areas F and G; we introduce the "dead area" I to ensure that in the vicinity of the central cross the path will continue to follow the same sort of circular arc that it does throughout the rest of the labyrinth; and finally we assume that the path ends at the point c0, so that the small semicircle J is also a "dead area." This reduction of the labyrinth walls to a series of circular arcs means that the path also reduces to a series of circular arcs (we assume that the path lies midway between the walls); and we can then calculate the total length of the path from a series of applications of the formula: circumference = 2π x radius (where π is the mathematical constant pi: 3.14159...). Further details of the calculation are given in footnote 2. The final result for the second-order labyrinth is that its path length is 114½ x π x its path-width (i.e. approximately 360 times).

This is only a special case of a more general result applying to any classical labyrinth constructed with the above "five-centre" method:

Theorem 1: If α is the order and P the path-width of a classical labyrinth, then its path length L is given by:

$$L = \tfrac{1}{2} \pi P (28 \alpha^2 + 48 \alpha + 21)$$

The first few cases are given in Table 1, where the path-width is taken as 1 unit. Column 6, "Factor," gives the values of $28 \alpha^2 + 48 \alpha + 21$. Table 1 also includes (column 4) the dimensions of the central cross/square; and (column 5) the diameter of each labyrinth at its maximum. As an example, if we were to construct a giant sixth-order labyrinth with one-yard wide pathway its maximum diameter would be 55 yards, and its path length 2069 yards, well over a mile.

Table 1: Path lengths of Classical Labyrinths

Order	Circuits	Walls	Square side	Major Diameter	Factor	Path Length
0	3	4	2	7	21	33
1	7	8	4	15	97	152
2	11	12	6	23	229	360
3	15	16	8	31	417	655
4	19	20	10	39	661	1038
5	23	24	12	47	961	1509
6	27	28	14	55	1317	2069

The Chartres-type (Medieval) Labyrinth

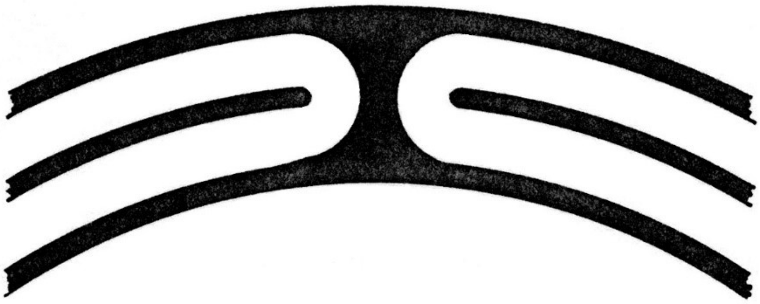
At first glance it might seem easier to calculate the length of this type (figure 3), with its strongly circular contours; but in fact, an exact formula involves trigonometric functions, as well as being both clumsy and inapplicable to variants such as the octagonal. Instead I propose the following general theorem:

Theorem 2:

If a labyrinth has pathways of constant width p, and walls of constant thickness w, overall area A, and total dead areas D (see footnote 3), then the path length L (measured along the centre of the pathway) is given by the approximate formula: L = (A - D) ÷ (p + w).

figure 3: the Chartres labyrinth

figure 4: the small 'dead areas' where the paths turn back



For more accuracy A should be replaced by A', the area inside the centre-line of the outer boundary walls, rather than inside the outer perimeter; and similarly, D replaced by D', measured inside the centre-lines of the walls enclosing the dead areas. In practice this distinction between centre-line and perimeter can often be ignored. Additionally, we can ignore the dead areas entirely, or estimate them roughly, provided they are small compared with the overall area. Applying Theorem 2 to the Chartres Cathedral labyrinth itself, using the dimensions quoted by Kern (see footnote 4), and ignoring the small dead areas where the paths turn back (see figure 4), we get:

$$L = \tfrac{1}{4} \pi (d^2 - c^2) \div (p+w) = 279 \text{ metres.}$$

This is smaller than the figure of 294 metres given by Kern, whose path length is measured along the "outer edges of the walls" rather than along the centre-line of the pathway. It is not clear to me exactly what path this represents, or whether it accounts for the difference. Matthews (*Mazes and Labyrinths,* 1922, p.59-60) on the other hand says "about 150 yards," which at approximately 137 metres is far too small – only about half the true figure.

Graeme Fyffe, London; 1989

Notes:

1. Terminology: a classical labyrinth is usually characterized by the number of its circuits, or of its walls. But the most basic parameter is the number of arcs per quadrant in the cross-dot-and-arc structure at its heart; this number I call the order. The order, the number of circuits, and the number of walls are easily deduced from each other. All three are listed in parallel in Table 1.
2. E.g.: area D comprises six quarter-circle portions of the full path, with radii 3½, 4½, 5½, 6½, 7½, 8½ times the path-width; so, the total length of path contributed by area D to the full path is: $\tfrac{1}{2} \pi (3\tfrac{1}{2}+4\tfrac{1}{2}+5\tfrac{1}{2}+6\tfrac{1}{2}+7\tfrac{1}{2}+8\tfrac{1}{2}) = 18\pi$. And similarly, for the other areas.
3. Definition: a dead area is one which the path never enters: it will generally consist of either an internal space entirely surrounded by walls; or a centre space which the path is considered not to pass through but to terminate on reaching.
4. Kern, Hermann. *Labyrinthe*. München, Prestel, 1982, p.225:

Overall diameter	d = 12. 45 metres (average)
Diameter of centre	c = 3.05 metres
Path width	p = 0. 34 metres
Wall thickness	w = 0.08 metres.